- Relative sensitivity

- Compare ^{1}H, ^{31}P

$$\gamma(^{1}H) = 267.522 \times 10^{6}\ s^{-1} \cdot T^{-1}$$

$$\gamma(^{31}P) = 108.394 \times 10^{6}\ s^{-1} \cdot T^{-1}$$

$$\frac{\gamma(^{1}H)}{\gamma(^{31}P)} \simeq 2.5$$

- Compare ^{1}H, ^{13}C

$$\gamma(^{1}H) = 267.522 \times 10^{6}\ s^{-1} \cdot T^{-1}$$

$$\gamma(^{13}C) = 67.283 \times 10^{6}\ s^{-1} \cdot T^{-1}$$

$$\frac{\gamma(^{1}H)}{\gamma(^{13}C)} \simeq 4$$

But, we also need to factor in the % of isotope in natural abundance:

Isotope	% NA	γ (10^6)
^1H	100	267.522
^{13}C	1.11	67.283
^{19}F	100	251.815
^{31}P	100	108.394
^{11}B	20	28.747
^{10}B	80	85.84

- Compare the combined influence of γ & NA. for ^1H & ^{13}C

$$\frac{\gamma(^1H)}{\gamma(^{13}C)} = 4 \quad ; \quad \frac{NA(^1H)}{NA(^{13}C)} = 100$$

Receptivity

- The strength of the NMR signal:

$$S_{max} \propto |\gamma^3 N|$$

nuclei

- The relative receptivity for $^1H/^{13}C$ is:

$$\frac{S_{max}(^1H)}{S_{max}(^{13}C)} = \frac{\gamma_{^1H}^3 N_{^1H}}{\gamma_{^{13}C} N_{^{13}C}}$$

$$= (4)^3(100)$$

$$= 6400$$

Now we reconsider polarization, N_α vs N_β

At 500 MHz (^1H), aka ~11.74 T

$$\frac{N_\beta}{N_\alpha} = \exp\left\{\frac{-\hbar\gamma B_0}{k_b T}\right\}$$

$$= \exp\left\{\frac{-(1.0546\times10^{-34}\ J\cdot s^{-1})(267.522\times10^{6}\ s^{-1}\cdot T^{-1})(11.74\ T)}{(1.3805\times10^{-23}\ J\cdot K^{-1})(298\ K)}\right\}$$

$$= \exp\left\{-8.051\times10^{-5}\right\}$$

$$\cong 0.99992$$

So, for every 100,000 spins (^1H nuclei)...

$$\frac{N_\beta}{N_\alpha} = 0.99992\ ;\quad N_\beta = (0.99992)N_\alpha$$

$$N_\alpha + N_\beta = 100{,}000\ ;\quad N_\alpha + (0.99992)N_\alpha = 10^5$$

$$(1.99992)N_\alpha = 10^5;\quad N_\alpha = 50002$$

$$N_\beta = 100{,}000 - N_\alpha = 49998$$

$$N_\alpha - N_\beta = 4$$

So, @ 11.74 T (500 MHz ^1H):

- Only 4 spins per 10^5 are net
- We need a lot of spins!
- In 0.8 ml of say H_2O there are

$$\rho_{H_2O} \cdot V$$

$$\left(\frac{1g}{1ml}\right)(0.8ml) = 0.8g$$

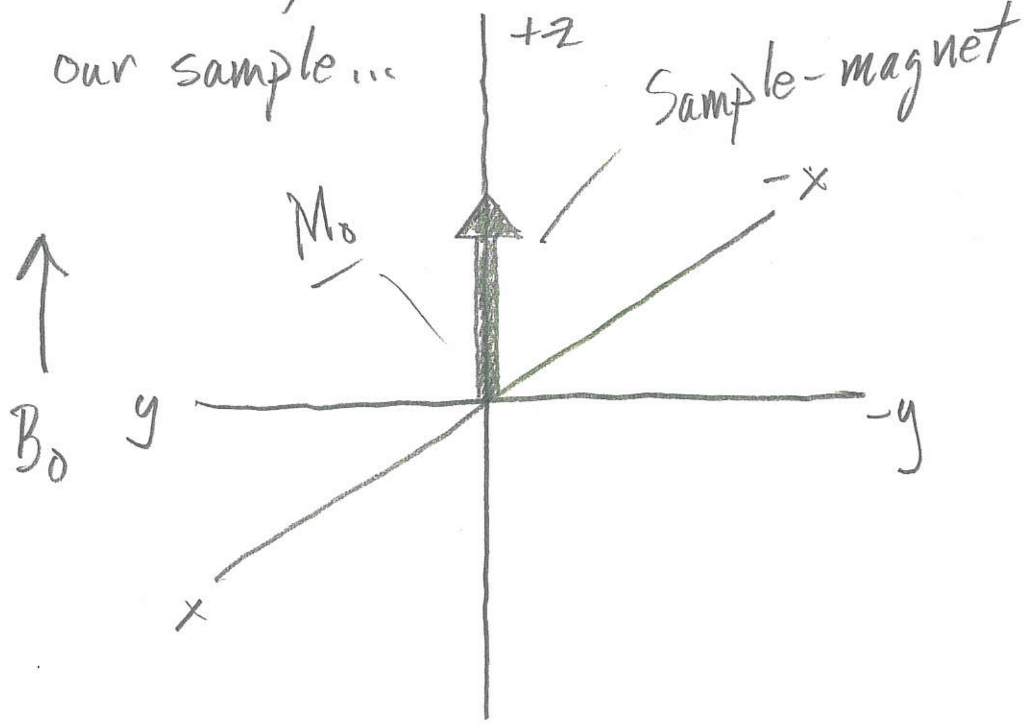
$$\# mol\ H_2O = \frac{0.8g}{18g/mol} = 0.028\ mol$$

$$\#\ \text{molecules}\ H_2O = N_a(0.028\ mol)$$

$$= (6.022 \times 10^{23}\ mol^{-1})(0.028\ mol)$$

$$= 1.67 \times 10^{22}$$

- Now then, sometime after we introduce our sample...



- Our sample is now a magnet!
- The phenomenon described above was named by our hero Felix Bloch,

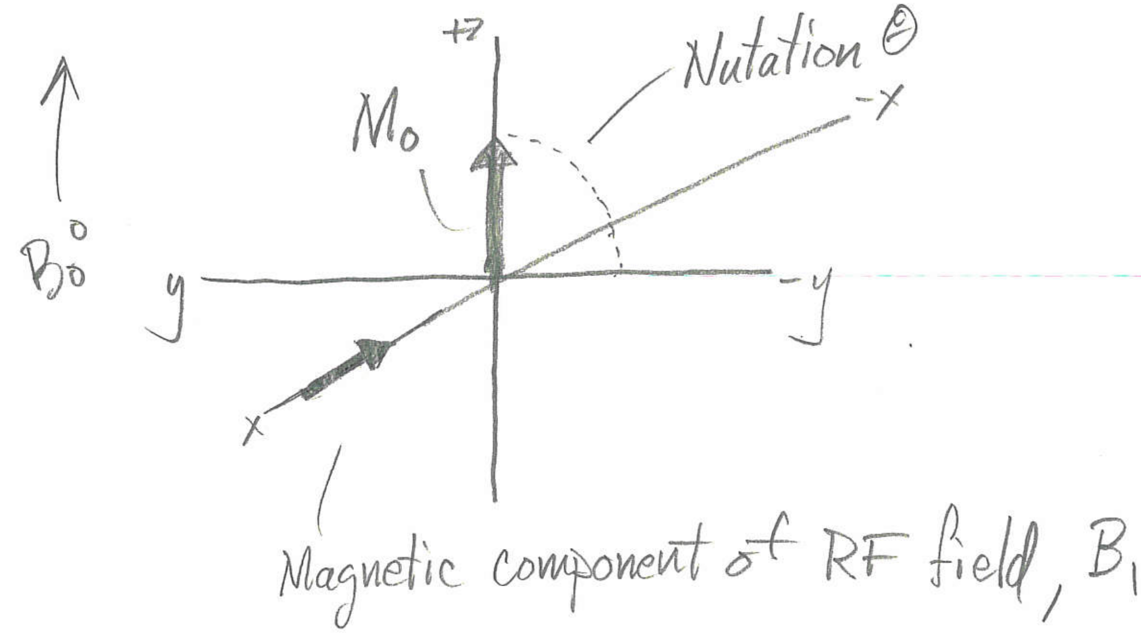
Nuclear Induction

F. Bloch
b. 1905
d. 1983
ETHZ (Debye & Weyl)
PhD (Heisenberg, Leipzig)

- The remainder of NMR technology, more or less, is concerned with perturbation or the induced nuclear magnetism.

- Most basic experiment:

Nutation - Detection of Free Precession



- The magnetic component of an applied radio-frequency (RF) field exerts a torque on M_0, causing nutation of M_0 in the direction of the $-y$-axis as predicted by the standard interpretation of the right-hand-rule...

- Non-standard right-hand-rule?!

$$\underbrace{\frac{d\vec{M}}{dt}}_{\substack{\text{Rate}\\ \text{of}\\ \text{Change}}} = \underbrace{\vec{M} \times \gamma \vec{B}_1}_{\text{Torque}}$$

- The nutation-θ thus depends on:
  1) The intensity of the magnetic component of the applied RF field, B_1
  2) The gyromagnetic ratio of the spins
  3) The duration during which the RF is applied

- Rotation of a magnetic moment around an applied magnetic field is predicted by the Larmor Theorem:

$$\vec{\omega} = -\gamma \vec{B}$$

- The nutation angle is

$$\theta = \tau_p \gamma B_1$$

- Thus, given a particular spin, e.g. ^1H, and a constant value for B_1, the nutation angle increases linearly with time.

- A characteristic parameter for NMR instruments is the τ_p such that $\theta = \frac{\pi}{2}$ (90°)

- Consider ^1H : 10μs RF duration (pulse!) $\tau_p(90°)$

$$\frac{\pi}{2} = (10^{-6}\,s)(267.522 \times 10^{6}\,s^{-1} \cdot T^{-1})\,B_1$$

$$B_1 = \frac{(1.5708)}{267.522 \cdot T^{-1}}$$

$$B_1 = 5.872 \times 10^{-3}\,T$$

$$= 58.72\,G \;\{ \times 100 \text{ B of Earth!}$$